Supplemental Material - Parameter Studies

A Lagrangian Method for Extracting Eddy Boundaries in the Red Sea and the Gulf of Aden

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1 Influence of Core Line

For several steps of our algorithm, the choice of the core line is of high importance As no unique definition for vortices exists, a set of methods is available to chose from. For an overview of vortex core line algorithms, we refer to the state of the art report by Günther et al. [1].

To explore the effects of varying core lines, we compare 4 different methods: Figure 1 shows them over time. Their general behavior is the same, while the lines vary greatly on a high-frequent scale.

Despite their major influence and difference between core lines, the results are very similar: In Figure 2 the same error graph was computed for the 4 core lines described above. The outer edges are very stable, only in the middle region different extremal values show up. They occur when particles move close to the core line: Small deviations of either line lead to high changes of the angle function α, even breaking its smoothness. However, after filtering out these regions and applying a threshold to them, all graphs result in the same boundary.

The stability of our algorithm to different cores increases with outgoing radius. This means, for any core line laying roughly central in the eddy area the result will be similar. Vice versa, if a core line moves outwards or even outside of the eddy, the results break. This is especially problematic for small eddies, since the error tolerance is significantly smaller.

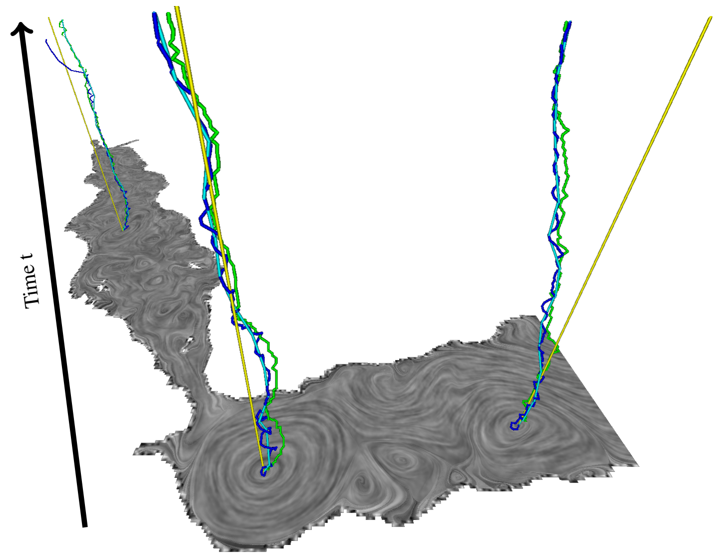

Figure 1: Different cores over time: A straight line seeded at the first critical point observed (yellow), the stream line core connecting all critical points over time (blue), an approximate stream line core connecting every 5$^{\text{th}}$ critical point (light blue) and the maxima of the SLA field (green).

2 Maximal Angle

Another parameter influencing the computation of the error function is the maximal angle we integrate over, set as α. The error graphs for different values of α are depicted in Figure 3. For very short values, we do not capture the long-term behavior of particles. When the outside flow close to the boundary is temporarily resembling that of the eddy, the error measure fails to capture the transition. This can be observed in Figure 3a: The present flow south of the eddy is very similar to the inside (compare for example Figure 1), so for lines moving only half a turn around the core no difference in behavior is notable. As a result, the boundary of the eddy is only defined by the coast line. When increasing the observed angle, edges begin to form in the south-west as particles formerly moving in parallel to the eddy are now drawn outwards. For higher values of α, these edges remain stable and become more pronounced. At the same time, most error values get higher overall. For the computation of error graphs throughout this paper we used $\alpha = 10$, slightly below two full turns. This corresponds to integrating over a maximum of 50 time steps (12.5 days).

3 Error Threshold

The final value to be set is the threshold we use to identify the boundary. Edges within the error graph tend to be very sharp, rendering the exact choice of value irrelevant. We have found thresholding values as low as 1.5 to obtain nearly the same results as values above 8. We chose a threshold of 6 for the final tubular structures extracted, while any value within $[3, 12]$ would have given only very slightly varying boundaries.

References

[1] T. Günther and H. Theisel. The state of the art in vortex extraction. *Computer Graphics Forum*, page to appear, 2018.

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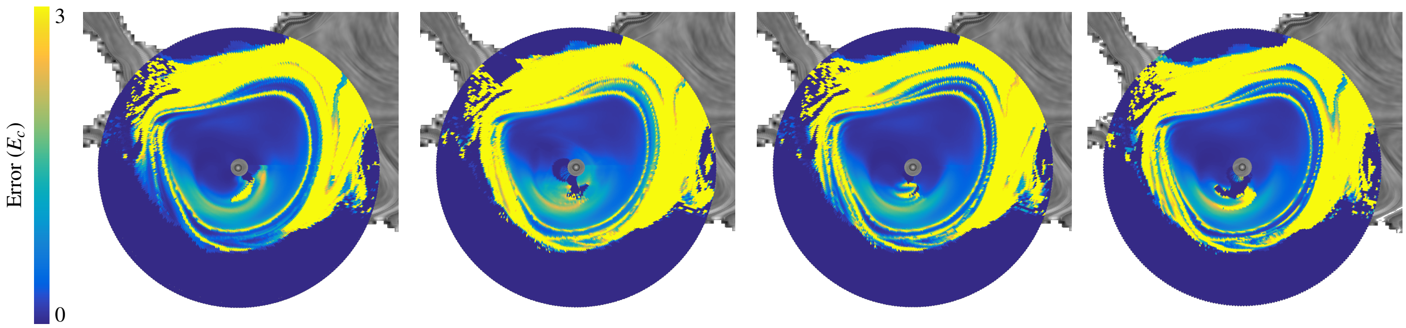
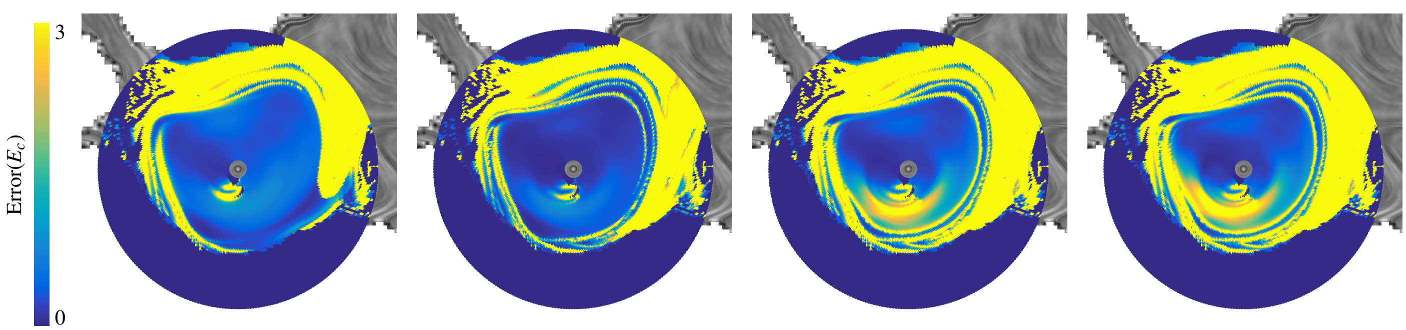


(a) Error using the straight core (b) Error using the stream core (c) Error using approximate stream core (d) Error using the SLA extrema core

Figure 2: The error graph E_c computed with different core lines. All graphs use an integration angle of 4π and seeding time 0. In the middle region different extremal values show up, hinting at particles moving very close to the core.



(a) Error for $\alpha = \pi$ (b) Error for $\alpha = 2\pi$ (c) Error for $\alpha = 3\pi$ (d) Error for $\alpha = 4\pi$

Figure 3: Error map E_c for different angle integration times α. For increasing values of α, more structures form.